Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

7. Location of maximum. Could we find a profit $f[x_1, x_2] = a_1 x_1 + a_2 x_2$ whose maximum is at an interior point of the quadrangle in figure 474? Give a reason for your answer.

```
d1 = ImplicitRegion[
  {x > 0, y > 0, y < -5/2 x + 30, y < (2.5 / -10) x + 7.75}, {x, y}]
```

```
ImplicitRegion[x > 0 && y > 0 && y < 30 - (5 x)/2 && y < 7.75 - 0.25 x, {x, y}]
```

```
ziz = Plot[{-(5 x)/2 + 30, -(2.5` x)/10 + 7.5`}, {x, 0, 20}, AspectRatio -> Automatic];
dip = Graphics[Polygon[{{0, 0}, {12, 0}, {10, 5}, {0, 7.5}, {0, 0}}]];
Show[ziz, dip];
```

The answer to the question is no, as demonstrated below. I changed the less-than-or-equals signs for strictly less-than, and it blew up. So no solution is available which is interior to the boundary lines.

```
Clear["Global`*"]

Maximize[{40 x + 88 y, 2 x + 8 y ≤ 60, 5 x + 2 y ≤ 60}, {x, y}]
{840, {x -> 10, y -> 5}}

Maximize[{40 x + 88 y, 2 x + 8 y < 60, 5 x + 2 y < 60}, {x, y}]
Maximize::wksol : Warning: there is no maximum in the region in which the objective
    function is defined and the constraints are satisfied; a result on the boundary will be returned. >>
{840, {x -> 10, y -> 5}}
```

9. What is the meaning of the slack variables x_3, x_4 in example 2 in terms of the problem in example 1?

Looking at example 1.

```
Clear["Global`*"]

Maximize[{40 x + 88 y, 2 x + 8 y ≤ 60, 5 x + 2 y ≤ 60}, {x, y}]
{840, {x -> 10, y -> 5}}
```

The above expression shows that the slack variables are unnecessary here, unlike problem 17, where I found them to be necessary. I guess if I knew that the problem would find x and y positive, I could leave out the slacks. And if it wouldn't solve, or the signs came out wrong, I could put them in.

11 - 16 Maximization, minimization
Maximize or minimize the given objective function f subject to the give constraints.

11. Maximize f = $30\,x_1 + 10\,x_2$ in the region in problem 5.

The region of problem 5: $-x_1 + x_2 \geq 0$; $x_1 + x_2 \leq 5$; $-2\,x_1 + x_2 \leq 16$

```
Maximize[{30 x + 10 y, -x + y ≥ 0, x + y ≤ 5, -2 x + y ≤ 16}, {x, y}]
```

$$\left\{100, \left\{x \to \frac{5}{2}, y \to \frac{5}{2}\right\}\right\}$$

13. Maximize f = $5\,x_1 + 25\,x_2$ in the region in problem 5.

```
Maximize[{5 x + 25 y, -x + y ≥ 0, x + y ≤ 5, -2 x + y ≤ 16}, {x, y}]
```

$$\left\{\frac{595}{3}, \left\{x \to -\frac{11}{3}, y \to \frac{26}{3}\right\}\right\}$$

15. Maximize f = $20\,x_1 + 30\,x_2$ subject to $4\,x_1 + 3\,x_2 \geq 12$, $x_1 - x_2 \geq -3$, $x_2 \leq 6$, $2\,x_1 - 3\,x_2 \leq 0$.

```
Maximize[{20 x + 30 y, 4 x + 3 y ≥ 12, x - y ≥ -3, y ≤ 6, 2 x - 3 y ≤ 0}, {x, y}]
```

```
{360, {x → 9, y → 6}}
```

17. Maximum profit. United Metal, Inc., produces alloys B_1 (special brass) and B_2 (yellow tombac). B_1 contains 50% copper and 50% zinc. (Ordinary brass contains about 65% copper and 35% zinc.) B_2 contains 75% copper and 25% zinc. Net profits are $120 per ton of B_1 and $100 per ton of B_2. The daily copper supply is 45 tons. The daily zinc supply is 30 tons. Maximize the net profit of the daily production.

```
Clear["Global`*"]
```

This took longer than it should have due to my befuddlement. In this analysis, I have x as the weight per day of B1, and y as weight per day of B2. The 120 and 100 profit is also on per weight basis, so those weight units cancel. The tons units don't need to show in the raw ore weight either, since they are included in the x and y, which stand on the other side of the equals sign. The **Maximize** expression was judged as unbounded by Mathematica until I put in the constraints for both x and y to be greater than or equal to zero (the slacks). The other thing that I should point out is that what I am trying to maximize is actually a function of two variables.

```
Maximize[{120 x + 100 y, 0.5 x + 0.75 y ≤ 45 ,
  0.5 x + 0.25 y ≤ 30, x ≥ 0, y ≥ 0 }, {x, y}]
```

```
{8400., {x → 45., y → 30.}}
```

19. Maximum output. Giant Ladders, Inc., wants to maximize its daily total output of large step ladders by producing x_1 of them by a process P_1 and x_2 by a process P_2, where P_1 requires 2 hours of labor and 4 machine hours per ladder, and P_2 requires 3 hours of labor and 2 machine hours. For this kind of work, 1200 hours of labor and 1600 hours on the machines are, at most, available per day. Find the optimal x_1 and x_2.

Here x and y will both be ladders per day. $\frac{L}{D}$P1 = x and $\frac{L}{D}$P2 = y.

```
Maximize[{x + y, 2 x + 3 y ≤ 1200, 4 x + 2 y ≤ 1600, x ≥ 0, y ≥ 0}, {x, y}]
```

```
{500, {x → 300, y → 200}}
```

21. Maximum profit. Universal Electric, Inc., manufactures and sells two models of lamps, L_1 and L_2, the profit being \$150 and \$100 respectively. The process involves two workers W_1 and W_2 who are available for this kind of work 100 and 80 hours per month, respectively. W_1 assembles L_1 in 20 min and L_2 in 30 min. W_2 paints L_1 in 20 min and L_2 in 10 min. Assuming that all lamps made can be sold without difficulty, determine production figures that maximize the profit.

The profit from lamps. Where x represents the profit from L1 lamp and y the profit from L2 lamp. Remembering to convert hours to minutes.

```
Maximize[
 {150 x + 100 y, 20 x + 30 y ≤ 6000, 20 x + 10 y ≤ 4800, x ≥ 0, y ≥ 0}, {x, y}]
```

```
{37 500, {x → 210, y → 60}}
```

Showing agreement with the text answer is the monthly profit in dollars, and the number of lamps produced and sold of the two types.